A Model with Free Particles Used for Numerical Simulation of Charpy Impact Test of Plastic Materials

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This paper presents a model for the numerical simulation of Charpy test, using together Smoothed Particle Hydrodynamics (SPH) Method and Finite Element Method. Numerical simulation of the mechanical tests require the use of special material models, so the material fracture to be also simulated. The new numerical method, SPH Method, in fact a version of the Free Particle Method, presents some important advantages among which no material failure criterion to be used. Our proposed model is a model combining the FEM and SPH Model: only the specimen is modeled only by particles. The numerical results are compared with the results using only FEM and finally with the experimental data. Avoiding the material models, in fact the failure criterion, by SPH method presents a great advantages because any material model (Plastic-kinematic, Johnson-Cook, Modified Johnson-Cook, Picewise linear plasticity etc.) involves knowledge about some material constants. This aspect is very important and in the same time, very difficult. The numerical model together with some theoretical fundamentals of the SPH Method could be more then an invitation for using the SPH Method, it could be an available model able to inspire the researchers in their work. The paper is finished after some conclusions.

Keywords: Charpy test, free particles, numerical simulation, FEM, SPH

For their properties, plastic materials are used more and more, even in those conditions where safety requirements must be full filled, in static and dynamic conditions. Plastic materials properties were essentially improved in last period and many plastic materials types are also developed. Using domains of plastic materials are very different, starting with the most important domains like aeronautics, healthy devices, electronics etc., until home and common devices.

Mechanical tests are the most important experiments to be passed by the plastic materials and perhaps Charpy and Izod tests are the most used. Such tests have to take place in accredited laboratories, in accordance with the recognized International Standard ISO/IEC 17025.

Just in these circumstances, numerical analysis or numerical simulation of the mechanical tests represents a researching way for improving of the plastic materials properties, which does not exclude the mechanical tests, but gives them more importance without an increase in costs.

As numerical analysis is concerned, this paper bring to us a numerical method, probably newest after the finite element method that practically is unused in our country: it is about Smoothed Particle Hydrodynamics (SPH) method - the most known and used version of the free particle method.

This method presents some important advantages comparatively with FEM, especially for its capability for avoiding the difficulties appearing owing to large deformation, but that method has also other advantages.

Among these advantages the possibility of describing the material rupture without using a special material model is one of the most important one.

In this paper, the theoretical fundamentals of SPH method are presented. The example chosen for demonstration is referring to the numerical simulation of Charpy test applied to a plastic material.

Fundamentals of SPH method

Smoothed Particle Hydrodynamics (SPH) method is a griddles Lagrangian technique which comes from astrophysics (1977). The SPH method was extended to fluid simulation, especially with free-surface (1992), but is more and more used in applied mechanics. Some substantial advantages of SPH method comparatively with Finite Element Method (FEM) occur.

These advantages are referring to the problem involving large deformations, then the lack of a grid allows some calculus facilities, including the contact modeling and not the last in an important order, the SPH method can describe the material rupture without using special material models. Therefore, using of SPH method does not require damage criterion and material constants of the adopted material model.

The SPH method belongs to the meshless methods, so the investigated domain is represented by a number of nodes, representing the particles of this domain, having their material and mechanical (mass, position, velocity etc.) characteristics. Each particle represents an interpolation point on which the material properties are known. The boundary conditions have to be imposed to some of particles, according to the problem analyzed, like in the case of finite element method.

The problem solution is given by the computed results, on all the particles, using an interpolation function. We can say that the fundamentals of SPH theory consist in interpolation theory; all the behavior laws are transformed into integral equations.

The kernel function, or smoothing function, often called smoothing kernel function, or simply kernel, gives a weighted approximation of the field variable (function) in a point (particle). Integral representation of a function f(x), used in the SPH method starts from the following identity:

$$f(x) = \int_{\Omega} f(x')\delta(x - x')dx' \qquad (1)$$

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where f is a function of a position vector x, which can be an one-, two- or three-dimensional one; $\delta(x - x')$ is a Dirac function, having the properties:

$$\delta(x-x') = \begin{cases} 1 \to x = x' \\ 0 \to x \neq x' \end{cases} \quad (2)$$

In equation (1), Ω is the function domain, which can be a volume, that contains the x, and where f(x) is defined and continuous. By replacing the Dirac function with a smoothing function W(x - x', h) the integral representation of f(x) becomes:

$$f(x) = \int_{\Omega} f(x') \cdot W(x - x', h) dx \quad (3)$$

where W is the smoothing kernel function, or smoothing function, or kernel function. The parameter h, of the smoothing function W, is the smoothing length, by which the influence area of the smoothing function W is defined (fig. 1).

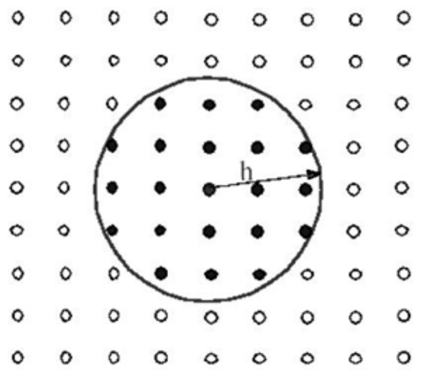

Fig. 1 Support domain of the kernel function

As long as Dirac delta function is used, the integral representation, described by equation (1), is an exact (rigorous) one, but using the smoothing function W instead of Dirac function, the integral representation can only be an approximation. This is the reason for the name of kernel approximation. Using the angle bracket, this aspect is underlined and the equation (3) can be rewritten as:

$$\langle f(x) \rangle = \int_{\Omega} f(x') \cdot W(x - x', h) dx \quad (4)$$

The smoothing function W is usually chosen to be an even one, which has to satisfy some conditions. The first condition, named normalization condition or unity condition is:

$$\int_{\Omega} W(x - x', h) dx' = 1 \quad (5)$$

The second condition is the Delta function property and it occures when the smoothing length approaches zero:

$$\lim_{h \to 0} W(x - x', h) = \delta(x - x') \quad (6)$$

The third condition is the compact condition, expressed by:

$$W(x - x', h) = 0 \quad \text{when} \quad |x - x'| > kh \quad (7)$$

where k is a constant related to the smoothing function for point at x, defining the effective non-zero area of the smoothing function, as the figure 2 shows.

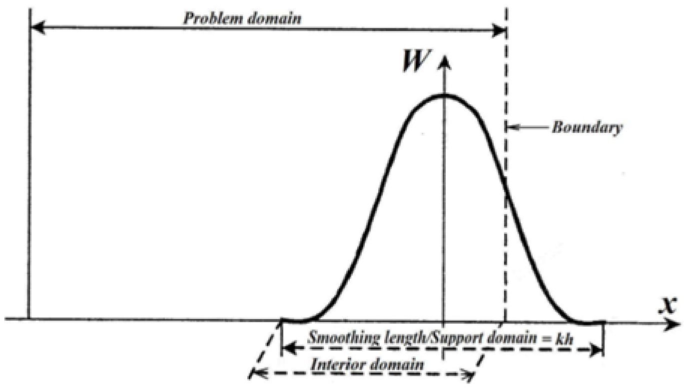

Fig. 2 Smoothing length

As the particle approximation is concerned, the continuous integral aproximation (4) can be converted to a summation of discretized forms, over all particles belonging to the support domain.

Changing the infinitesimal volume dx' with the finite volume of the particle ΔV_j, the mass of the particles m_j can be written,

$$m_j = \Delta V_j \rho_j \quad (8)$$

and finally, relation (3) becomes:

$$\langle f(x) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W(x - x_j, h) \quad (9)$$

The particle approximation of a parameter, described by a function, for a particle i, can be expressed by,

$$\langle f(x_i) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij} \quad (10)$$

where, W_{ij} is the kernel function.

$$W_{ij} = W(x_i - x_j, h) \quad (11)$$

So, concluding and in a synthetic presentation, the most important requirements of a kernel function are:

- the smoothing function has to be *normalized* over its support;
- the smoothing function has to be *compactly supported;*
- the smoothing function has to be *positive* for any point at x' within the support domain;
- the smoothing function value has to be *monotonically decreasing* with the increase of the distance away from the particle;
- the smoothing function value has to satisfy the *Dirac delta function condition* as the smoothing length approaches to zero;
- the smoothing function value has to be an *even function* (symetric).

The literature presents different smoothing function (also called smoothing kernel function, smoothing kernel, or kernel). Theoretically, any function having the properties presented above, can be employed as SPH smoothing function.

Graphical representation of the most used kernel functions until now is presented in the figure 3.

The Ls-Dyna program uses a cubic B-spline kernel function, in the form given by relation (12), where $s = r/h$, n is the number representing the spatial dimension and α is a constant which has the value: 2/3, $10/7\pi$ or $1/\pi$, depending on the space dimension.

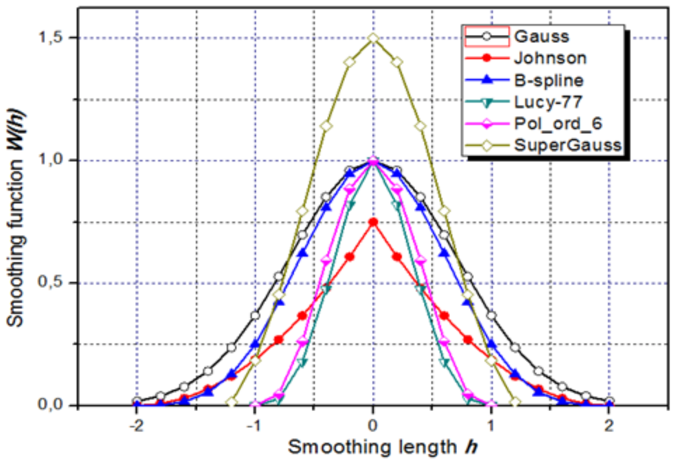

Fig. 3. Graphical representation of different kernel functions

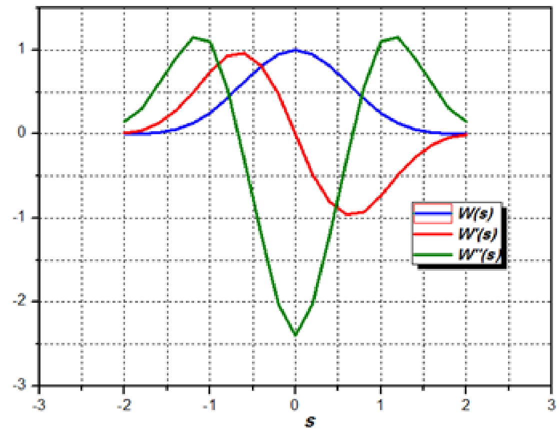

Fig. 4. Graphical representation of B-spline kernel function and its derivates

$$W(s,h)=\frac{\alpha}{h^n}\begin{cases}\left(1-\frac{3}{2}s^2+\frac{3}{4}s^3\right)\leftarrow 0\le s<1\\ \frac{1}{4}(2-s)^3\leftarrow 1\le s<2\\ 0\leftarrow s\ge 2\end{cases} \qquad (12)$$

The graphical representation of this smoothing function and its derivatives (first and second) can be seen in the figure 4.

A smoothing length too small (not enough particles in the support domain) influence on the calculus efficiency and also the accuracy, this going down. A smoothing length too large all the particle properties may be smoothed out and finally the accuracy will be a low one.

The best way seems to be a variable smoothing length according to calculus and accuracy efficiency. So, many ways already exist for a dinamically evolving of *h*, for geting a suitable number of the neighboring particle, which to remain relatively constant.

The simplest approaching is that the smoothing length to depend on the average density. From this point of view, the literature proposed the following relation:

$$h=h_0\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{d}} \qquad (13)$$

where h_0 and ρ_0 are the initial smoothing length and density respectively; *d* is the number representing the space dimension (1D, 2D or 3D, or simply 1, 2, or 3).

Numerical Model of Charpy Impact Test

For evaluation of the mechanical properties, in dynamic condition, of the plastic materials and others as well, the Charpy and/or Izod test is used. The free particle method in SPH version is very efficient for simulation of these tests, using only fundamental information about a material, namely: Young modulus, density and yielding stress. The material model can be the simplest one or the most sophisticated that take into account a fracture criterion, the influence of strain rate and the consolidation phenomenon.

For numerical simulation, all requirements regarding the geometry of the striker (hummer), of the specimen and its supporting were fulfilled, in other words all requirements coming from standards ISO 179-1 and ISO 179-2 have been met. The numerical simulation of Charpy test, presented here, used a notched specimen. In the figure 5 we can see the numerical model: a) frontal view and b) a 3-D view.

The striker is modeled with 1660 SOLID finite elements (FE) having variable dimensions, from 0.05 mm in the contact zone, to 2 mm in the far zone of the contact. As we can see, only a part of the striker is considered - that part which has a contact with the specimen. The material

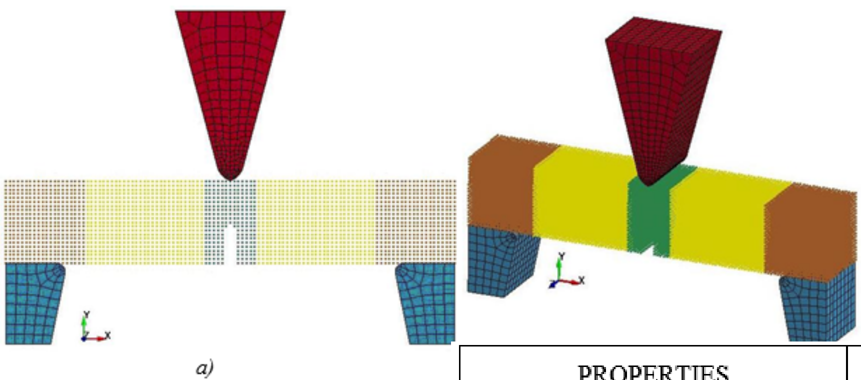

Fig. 5 Numerical model with free particles

Table 1
MATERIAL PROPERTIES OF THE SPECIMEN

PROPERTIES	TEST METHOD	UNITS	VALUES
Density	ISO 1183	g/cm^3	1.41
Tensile Modulus	ISO527-1 / 527	MPa	2800
Yield Stress	ISO527-1 / 527-2	MPa	63
Nominal Strain at Break	ISO527-1 / 527-2	%	33
Charpy Notched Impact Strength	ISO 179-2	kJ/m^2	8

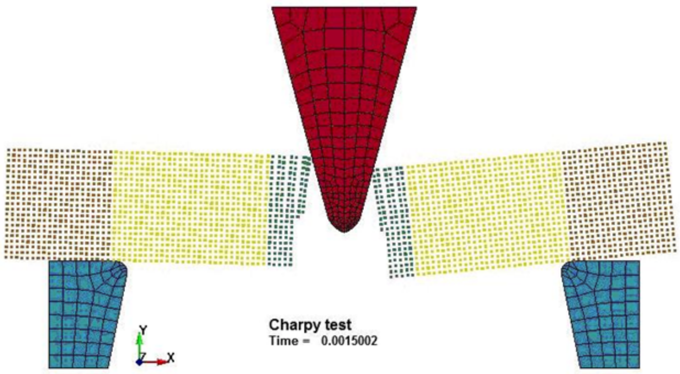

Fig. 6 Final state of specimen

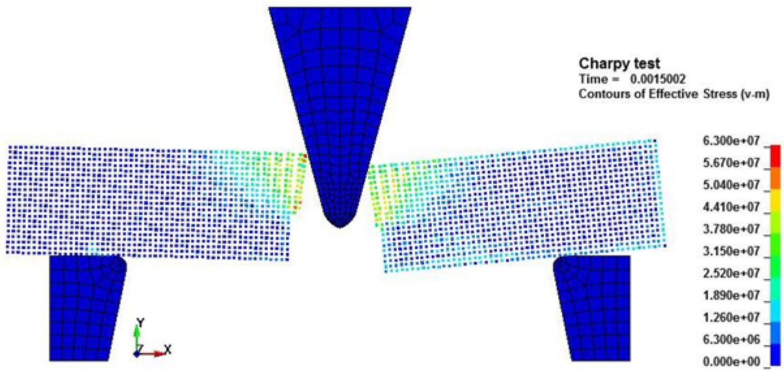

Fig. 7 Equivalent von Mises stress field at the end of analysis period

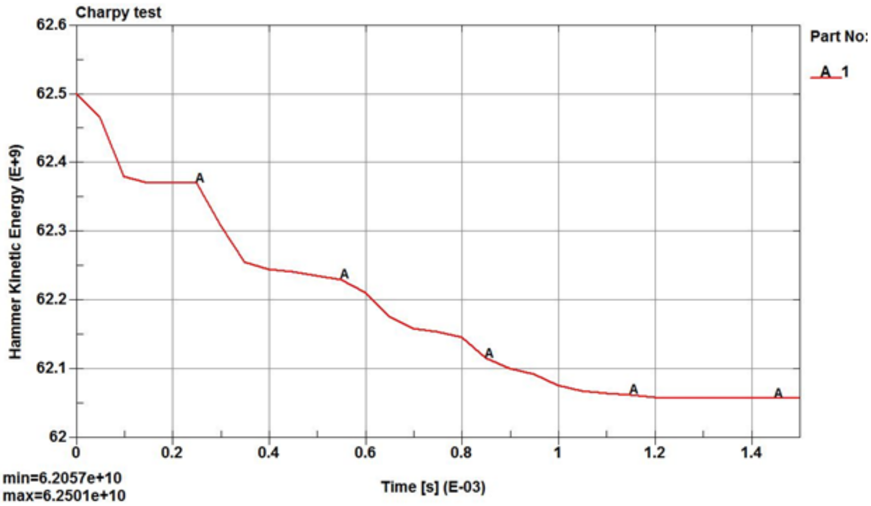

Fig. 8 Time evolution of the hammer kinetic energy

model used for striker is rigid material model - a choosing which makes easier the computer calculus. This is a right choosing and by the reason that striker material has a much more strength comparatively with a plastic material.

The material density is calculated from requirement that the striker volume adopted to lead us to a realistic mass value of an Charpy hammer. We adopted a density of 3.057 g/mm^3, resulting a mass of 5 kg. For a striker kinetic energy of about 60 J, the impact velocity was 5 m/s.

The specimen has the dimensions 10 x 10 x 55 mm with an *U* notch, which complies with all standards. So, the fracture section aria is 50 mm^2. The specimen is modeled by 48363 particles, uniformly distributed in volume, with an internodal distance of 0.50 mm.

The specimen material is a Mitsubishi Engineering Plastics, named lupital Acetal F10; the main properties of this plastics are presented in the table 1.

Those three different colored portions of the specimen (fig. 5) represent three identical parts of the specimen and this organization is a calculus trick for a smaller computer time.

The restrictions imposed to the specimen consist in zero Z-displacement of the nodes placed in the longitudinal symmetry plane (xOy).

Specimen is supported on an anvil as the figure 5 presents.The anvil also meet standard requirements and it is modeled with 816 SOLID finite elements.

Adopted analysis time resulted from condition that the striker to pass through specimen, this being broken.

Results

The final state of numerical model is presented in the figure 6 and 7, where the von Mises equivalent stress field is represented.

We used a unit measure system having the fundamental units g/mm/s. In this circumstances, the kinetic energy, represented in the figure 8, has the maximum values 62.5 Nm and the minimum value 62.057 Nm. In the Charpy test, this energy variation (0.443 Nm or J) represents the energy absorbed by the specimen during the fracture process. The calculus of Charpy test result can be obtained imediately:

$$KCU = \frac{0.443 \cdot 10^{-3}}{50 \cdot 10^{-6}} \frac{kJ}{m^2} = 8.86 \frac{kJ}{m^2} \quad (14)$$

As it is recommended, this energy has to be less then 10% of maximum hammer kinetic energy (6.25 Nm). But

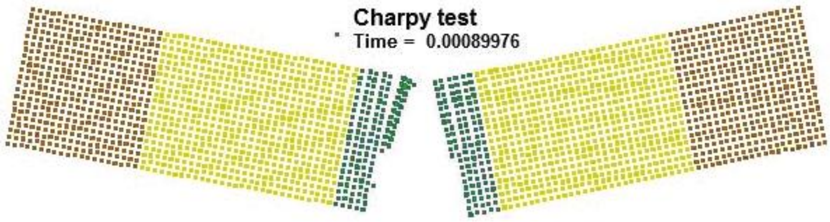

Fig. 9 The specimen state at the moment of rupture

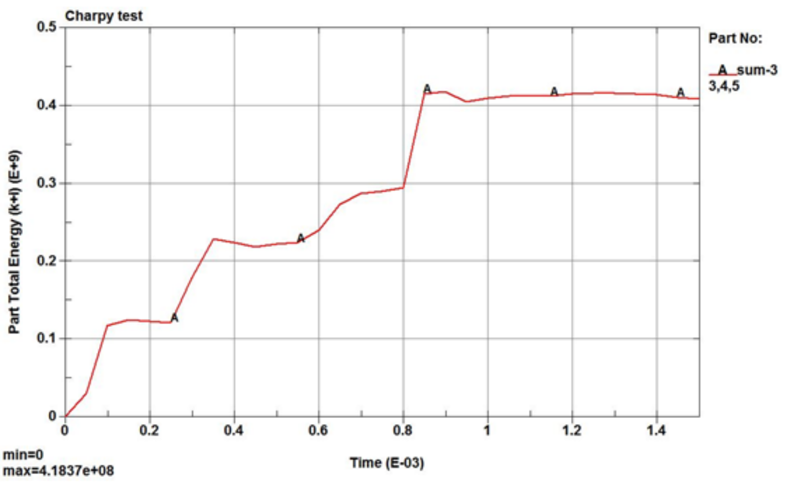

Fig. 10 Time evolution of the specimen total energy (kinetic & internal)

the result of relation (14) has not enough accuracy because the moment of rupture finishing is not the moment of our analysis. By graphical post-processing the rupture moment can be determined with accuracy. Figure 9 shows this moment.

From the file of striker kinetic energy, for the time 0.00089976 s, the value of kinetic energy is 62.099 J. Using this value in relation (14), the Charpy test result, KCU has the value 8.04 kJ/m^2, a value very closed with the manufacturer value.

With very good results, instead of striker kinetic energy, we can use the specimen total energy, represented in the figure 10. All energy types of the specimen come from the striker kinetic energy.

Conclusions

The appearing of this new numerical method, almost unknown and unused in our country, open a new way of numerical analysis in applied mechanics.

Using numerical simulation of the classical material testing can bring more accuracy in experiment, can make experimental investigation chipper but more efficient.

SPH modeling allow us to obtain good results just in the case of poor information about a material (without material constants of the material models).

The moment of total fracture of the specimen can be determined and so, we could have better information about material dynamic behavior.

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